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23BMT451

UG PROGRAM (4 YEARS HONORS) WITH SINGLE MAJOR
AT THE END OF FOURTH SEMESTER
MATHEMATICS-RING THEORY & PROBLEM SOLVING SESSIONS (Minor)
(w.e.f. Admitted Batch 2023-24)

Time: 3 Hours **Maximum: 70 Marks**

Section–A

Answer any Five Questions. **5x4=20**

1. Show that a ring R has no zero divisors if and only if the cancellation laws hold in R.
2. Show that the intersection of two subrings is a subring.
3. Give an example of a ring which is not a principal ideal ring.
4. Show that the homomorphic image of commutative ring is commutative.
5. Find the factors of $x^4 + 4$ in $Z_5[x]$
6. Show that the characteristic of a Boolean ring is 2.
7. If U is an ideal of the ring R and a, b $\in R$ then prove that a+U = b+U $\Leftrightarrow a - b \in U$
8. If f is a homomorphism from a ring R into a ring S then Ker f is an ideal of R.

Section– B

Answer All Questions. **5x10=50**

9. a) If p is a prime then prove that Z_p, the ring of integers modulo p, is a field.

(Or)

b) Show that a commutative ring R with unity is a field if and only if (0) and R are the only ideals of R.

10. a) Show that the necessary and sufficient condition for a nonempty subset S of a ring R to be a subring of R are i) $\forall a, b \in S \Rightarrow a - b \in S$, ii) $\forall a, b \in S \Rightarrow ab \in S$

(Or)

b) Show that the union of two ideals of a ring R, is an ideal of R if and only if one is contained in the other.

11. a) Show that the ring of integers is a principal ideal ring.

(Or)

b) Let S be an ideal of a ring R. Then show that

i) If R is commutative then R/S is commutative

ii) If R contains unity then R/S contains unity.

12. a) State and prove fundamental theorem of homomorphism of rings

(Or)

b) Show that an ideal S of a commutative ring R with unity is maximal if and only if the residue class ring R/S is a field.

13. a) State and prove Division Algorithm theorem in polynomials

(Or)

b) If F is a field, prove that every ideal in F[x] is a principal ideal.

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23BMT452

UG PROGRAM (4 YEARS HONORS) WITH SINGLE MAJOR
AT THE END OF FOURTH SEMESTER
MATHEMATICS-INTRODUCTION TO REAL ANALYSIS & PROBLEM SOLVING SESSIONS
(Minor)

(w.e.f. Admitted Batch 2023-24)

Time: 3 Hours **Maximum: 70 Marks**

Section–A

Answer any Five Questions. **5x4=20**

1. Show that $lim \sqrt{\frac{n+1}{n}} = 1$
2. Test for the convergence of $\sum \left(1+\frac{1}{n}\right)^{-n^2}$
3. If $f(x) = \frac{x^5-2^5}{x-2}, x \neq 2$ is continuous at $x = 2$, then define $f(2)$.
4. Find c of Cauchy's mean value theorem for f (x) = $x^2$and g(x) =x^3 in [1, 2]
5. If $f(x) = k \; \forall x \in [a,b]$ where k is a real number, show that $f \in R[a,b]$
6. Show that every convergent sequence is bounded.
7. State and prove Leibniz test.
8. Prove that the function defined by

$$f(x) = \begin{cases} 0 \; when \; x \in Q \\ 1 \; when \; x \in R - Q \end{cases}$$ is not integrable on any interval of R

Section– B

Answer All Questions. **5x10=50**

9. a) Test for the convergence of $1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+ \ldots$

(Or)

b) Show that the $\{s_n\}$ defined by $s_1 = \sqrt{2}, \quad s_{n+1} = \sqrt{2s_n}$ converges to 2

10. a) State and prove D'Alembert's Ratio Test.

(Or)

b) Test for the convergence of $\sum \frac{(-1)^n}{n}\left(1+\frac{1}{2}+\frac{1}{3}+ \ldots\ldots + \frac{1}{n}\right)$

11. a) Let f: [a,b]→ R. If f continuous on[a,b] and f(a), f(b) have opposite signs then show that there exists $c \in (a,b) \ni f(c) = 0$

(Or)

b) Examine the continuity of the function f defined by $f(x) = |x| + |x - 1|$ at $x = 0$ and 1

12. a) Verify Rolle's theorem for the function $f(x) = (x - a)^m (x - b)^n$ in [a, b] when m and n being positive integers.

(Or)

b) State and prove Lagrange's mean value theorem.

13. a) If $f : [a, b] \rightarrow R$ is continuous on $[a, b]$ then show that f is R – integrable on $[a, b]$

(Or)

b) Show that a bounded function f: [a, b]→ R is Riemann integrable on [a, b] if and only if for each $\varepsilon > 0$ there exists a partition P of [a, b] such that $0 \leq U(P,f) - L(P,f) < \varepsilon$